WorkKeys®

TARGETS FOR INSTRUCTION

Applied Mathematics

ACT®

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A copy of the full *Code* may be obtained free of charge from ACT Customer Services, P.O. Box 1008, Iowa City, Iowa 52243-1008, 319/337-1429.

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Applied Mathematics Skills

Applied Mathematics is the skill people use when they apply mathematical reasoning and problem-solving techniques to work-related problems. Solving mathematical problems in the workplace can differ from solving problems in the classroom. While the math skills needed are the same, math problems in the workplace are not usually laid out neatly in a textbook format. Instead, the employee may be responsible for identifying and locating the necessary information (e.g., on a cash register, price tag, or catalog) as well as knowing what to do with that information. It is, therefore, critical to strengthen each learner's core mathematics skills and to develop his or her repertoire of problem-solving strategies. Individuals possessing these Applied Mathematics skills will be able to successfully tackle novel situations involving mathematics problems in the workplace.

The *Applied Mathematics Targets for Instruction* is not intended to be a guide on how to teach math. Instead, this *Target:*

■ Describes the Applied Mathematics skills encompassed by each of the five hierarchical levels.

■ Emphasizes the types of mathematics tasks that are encountered in the workplace and modifications of teaching methods that correspond to these differences. For example, there is a special focus on calculators because of their common use in the workplace.

■ Suggests specific strategies that can be used as they are or be adapted to a variety of teaching or training situations to connect the skills to workplace situations.

There are five levels of proficiency in the Applied Mathematics skill scale, ranging from Level 3, the least complex, to Level 7, the most complex. These levels were developed focusing on two main criteria:

♦ the types of **mathematical operations** employees must perform, and

♦ the form and order in which employees receive the information; that is, the **presentation** of the information.

The skills at the lowest level involve using whole numbers and some decimals in basic math operations: addition, subtraction, multiplication, and division. As the levels progress, the math operations involve more steps. Furthermore, they include decimals and fractions, conversion of units, averaging, calculating area and volume, and ratios.

As the complexity of the levels increases, the presentation of the information becomes more of a barrier to problem solving. The wording becomes ambiguous, the presence of unnecessary information is more likely, and pertinent information is less obvious. Regardless of skill level, most of these problems will involve one or more of the following applications:

Quantity

Employees often need to determine the number of items sold, produced, or purchased, or to figure totals on a per-unit basis.

Money

Working with monetary units is a central part of business and is tangential to virtually every job, if in no other way than to understand a paycheck. Tasks involving monetary units include figuring sales, costs, wages, and expenses.
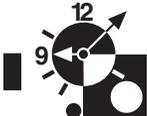
Time

Some tasks involve figuring elapsed time. Other problems are also frequently figured in terms of time (e.g., production, sales, costs, distance, area). In many of these tasks, employees must be familiar with conversion of time units.

Measurement

Calculating distance, area, weight, and volume is crucial to most work situations. Again, employees must be familiar with conversions, as well as the appropriate degree of accuracy needed for different situations.

Proportions and Percentages

Proportions can be used in many tasks that require making predictions (e.g., if this is the amount for X units, how much is needed for Y units?). Percentages are used in the workplace to calculate commissions, discounts, taxes, price increases, changes in sales, and wage changes.

Averages

Many records in the workplace are expressed in terms of averages (e.g., those involving sales records, wages, costs, hours worked). These averages become tools in the decision-making processes of the business.

Many math problems found in the workplace combine two or more applications: What **quantity** can be produced in a specified **time?** What **distance** can be traveled in a particular **time?** What is the **average** cost in terms of **money?** A common combination of applications is finding the *best deal,* which requires employees to perform various calculations and then compare the results in terms of relative cost, savings, etc. Examples of appropriate applications for each level will be given later in this *Target.*

Strategies for Teaching Applied Mathematics Skills

General strategies used to teach mathematics are applicable in preparing learners for the workplace. This *Target* seeks to expand those strategies by pointing out some work-related differences in the content and cognitive strategies required, by identifying appropriate materials, and by suggesting activities that may be particularly useful.

When choosing materials to help learners improve their Applied Mathematics skills, you should look for problems

- ♦ emphasizing the skills appropriate for the level the learners are trying to achieve;

- ♦ set in a work environment involving the applications described at that level; and

- ♦ with presentations that are appropriate for the level the learners are trying to achieve.

You may want to develop problems to simulate workplace applications. You may also want to contact local businesses for actual examples or have learners bring in examples from their own jobs. As often as possible, present these problems to the learners in the same way they would be presented in the workplace.

Working With Adult Learners

Since many WorkKeys learners are adults, there are some useful points concerning adult learning to keep in mind. Adults are usually motivated to learn something primarily because they believe it will be useful. You will therefore want to be sure that learners understand and appreciate the connections between Applied Mathematics skills and the workplace.

Adults especially need to be able to relate new material to something connected with what they already know. Otherwise they are less likely to retain or use the new material. They may also have less self-esteem in the classroom than younger learners. You

can respect this by giving them ample opportunities to practice a skill on their own successfully before demonstrating the skill in front of others, or by having them work in small groups or teams to learn collaboratively.

Although there is no universal agreement among educators on correct approaches to teaching adults in comparison to children, many feel that there are a number of good teaching practices that should especially be used in adult education. In "Using Adult Learning Principles in Adult Basic and Literacy Education," Susan Imel summarizes the recommendations from several sources:

- ■ **Involve adults in program planning and implementation.** This practice can inform the instructor more completely about the learner's previous educational experiences, relate the material to the learner's present needs, and improve motivation.

- ■ **Develop and/or use instructional materials that are based on students' lives.** Again, the focus is on relating learning more directly to the learner's experience. It is especially appropriate to use workplace documents and problems from the learner's work environment when addressing the generic skills assessed by the WorkKeys tests.

- ■ **Develop an understanding of learners' experiences and communities.** Although individualizing instruction has great benefits, it is important to keep the learner's community background and daily life in view. This can help you to understand their motivations and problems and can provide material and help to identify strategies that they can relate to.

- ■ **Incorporate small groups into learning activities.** Small group work has been used successfully with all ages. For adults, this approach can provide peer support, a context more similar to those where they actually practice mathematics skills—that is the workplace, home, and other daily settings.

Whatever the level of the learner or the length of the program, it is important to remember the following guidelines:

♦ Allow enough time to effect a permanent increase in all skill levels by incorporating adequate practice to establish solid competence.

♦ Be sure each learner is clearly aware of his or her own goals and of the relationship between those goals and job qualifications.

♦ Use pretests to motivate learners and avoid time-consuming reteaching of skills they have already mastered. Posttesting or observing of learners using a checklist may be useful for evaluating their mastery of skill levels.

♦ Present the instruction sequentially; learners should master each step before going on to the next.

♦ Be sure that learners demonstrate the prerequisite skills for each level before continuing with the instruction.

The Skill Scale and Assessments

WorkKeys assessments simulate the requirements of the workplace to the maximum degree possible given the requirements of a large-scale, standardized assessment. As a result, WorkKeys is not geared toward any particular age group but, instead, targets the requirements of the workplace. The WorkKeys system provides information to instructors and trainers who can then help individuals improve their workplace skills.

WorkKeys assessments are based on common skill scales that are divided into hierarchical levels. The scale used to assess each skill is identical to the one used in the WorkKeys profiling of that skill. Profiling determines the skills and skill levels needed to perform successfully in particular jobs or occupations. In a WorkKeys profile, each level designation indicates

the level of that WorkKeys skill required to adequately perform the specified job or occupation. The same scale is also used to describe that skill in the corresponding *Targets for Instruction.* This common scale enables instructors to use the assessments, the profiling component, and the *Targets for Instruction* to help learners prepare for the jobs or occupations of their choice.

The number of levels and the range of the levels vary from skill to skill. For example, the skill scale for the Listening skill has levels ranging from 1 to 5, while the scale for the Applied Technology skill has levels ranging from 3 to 6. This variation reflects the levels of each skill that employers want tested. Level 3 in one skill is not necessarily equal to Level 3 in another skill.

For example, Figure A shows how the WorkKeys system matches the skills of a particular individual with the skill requirements of a particular job. The individual whose skill profile is shown may be currently employed in or interested in a particular job that requires Level 4 Listening skills, Level 4 Locating Information skills, and Level 6 Applied Mathematics skills. The individual's WorkKeys assessment scores show that he or she has achieved Level 4 Listening skills, Level 5 Locating Information skills, and Level 5 Applied Mathematics skills. The assessment results show that this individual needs to improve his or her Applied Mathematics skills in order to match the skill requirements of the job.
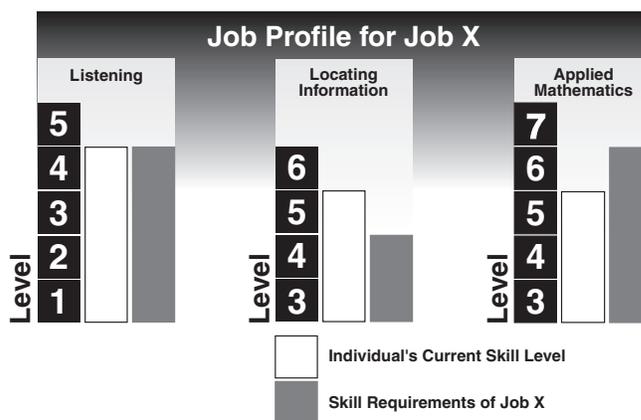

Figure A

The WorkKeys *Targets for Instruction* assist you in helping learners improve by focusing on the skills needed at each level of the WorkKeys skill scale. For the individual described in Figure A, this *Target* identifies the Applied Mathematics skills needed to perform at Level 6. It then suggests how to help the learner improve his or her skills from Level 5 to Level 6. Remember that it is not the purpose of the *Targets for Instruction* to teach the exact material that is on the tests but, rather, to guide instructors as they help learners build the skills that characterize each level of the skill scale.

The occupational profiles can be used to inform individuals about the generalizable workplace skills that they are likely to need in selected occupations. The occupational profiles may also serve as a starting point for a discussion about standards and/or requirements for entering or exiting a training program.

A comprehensive list of the occupational profiles that have been completed by WorkKeys job profilers is available at http://www.act.org/workkeys/profiles/occuprof/index.html.

Format and General Strategies of the Assessment

The WorkKeys *Applied Mathematics* assessment measures the examinee's skill in applying mathematical reasoning and problem-solving techniques to work-related problems in a format suitable for large-scale, standardized administration. The math items are presented in the form of story problems involving a workplace context. The examinee is placed in the role of an employee who must solve an Applied Mathematics problem to complete his or her workplace tasks. These problems are arranged in order of increasing complexity so that each test form begins with the least complex problems and ends with the most complex. The examinees' responses are *dichotomously scored* (that is, are either correct or incorrect). Answers left blank are treated as wrong answers; there is no penalty for guessing.

In most workplace situations, employees have access to calculators and conversion tables. Likewise, on the *Applied Mathematics* assessment, examinees may use calculators, and they are given a table of basic formulas and unit conversions to use (see Appendix A).

There are several equivalent forms of the *Applied Mathematics* assessment. Each form contains multiple-choice items at five levels of difficulty. The levels range from Level 3, which is the least complex, to Level 7, which is the most complex. Note that Level 3, as its designation implies, contains more advanced problems than the simplest mathematical operations an individual could perform. Likewise, there are more complex mathematical problems than those presented at Level 7 of this assessment.

In developing the WorkKeys system, educators, employers, and ACT staff identified Level 3 skills as comprising the lowest level of mathematics for which employers would likely want assessment results. If an examinee receives a score below Level 3, you should investigate whether this score represents strictly an Applied Mathematics deficiency, or whether an intervening factor influenced test performance. For example, external factors such as health, a distraction during the administration process, lack of motivation, English as a second language, visual impediment, learning disability, or a high stress level may have impeded test performance.

The accuracy and appropriateness of the items have been reviewed and endorsed by content and fairness experts. The assessment forms are constructed so that the occupations described are varied. Care has been taken to ensure that the assessment items are as realistic as possible and that the content of the tasks is accurate. The items focus on situations that might actually be encountered in the workplace. The tasks contain enough detail to create a realistic workplace context, but not so much detail that the assessment is job specific. The items at each level are comparable to one another across the different forms.

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SKILL LEVELS

Applied Mathematics

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3 Description of Level 3 Skills

Level 3 tasks can easily be translated from a verbal situation to a math equation. All the needed information is presented in a logical order and there is no extraneous information. Individuals with Level 3 Applied Mathematics skills can:

- **Solve problems that require a single type of mathematics operation.** They add or subtract either positive or negative numbers (such as 10 or –2). They multiply or divide using only positive numbers (such as 10). In the workplace, problems at this level involve using basic operations to find, for example, the number of items per carton, the total number of units sold, or the number of units left in stock. Positive and negative numbers are used to indicate change and the direction of the change, such as in temperature readings and in stock market prices.

- **Change numbers from one form to another.** For this they use whole numbers (such as 10), fractions (such as $\frac{1}{2}$), decimals (such as 0.75), or percentages (such as 12%). For example, they can convert $\frac{4}{5}$ to 80% or convert 35% to 0.35.

- **Convert simple money and time units.** In the workplace, hours need to be changed to minutes or dollars to cents and vice versa.

Moving to Level 3 Skills

A learner who has difficulty at Level 3 may become overwhelmed by even approaching a math problem. For the older teen or adult who is still at this level, these feelings can be compounded by embarrassment. Many learners at all levels are daunted by problems that seem too complex for them. To break this pattern, the learner should develop strategies to divide the problem into manageable segments. To help learners attain Level 3, have them work with problems that are presented in different ways and help them develop a problem-solving strategy that can be applied to all tasks at all levels.

UNDERLYING ASSUMPTIONS

The learner needs certain background knowledge and supporting skills to succeed at Level 3. You may want to evaluate the following areas and provide instruction where appropriate.

Background Knowledge

Learners should be able to recognize certain common symbols, such as the dollar sign and percent sign; make basic time conversions; and convert between dollars and cents.

Background Skills

In the workplace, an individual may perform computations manually, but calculators are often used and can improve accuracy. Therefore, you may want to evaluate your learners' proficiency with calculators. If there are deficiencies, you may want to develop training or review sessions targeting basic calculator skills. At the least, learners should be adept at using the four basic function keys (+, -, ×, and ÷) and should understand the purpose of the "Clear" key.

PRESENTATION OF PROBLEMS

Often mathematics textbooks present word or story problems in neat, comprehensive paragraphs. They provide all of the necessary information to solve the problem, and word choices may help the learner select the appropriate processes. However, individuals on the job encounter mathematics problems in a variety of presentations using data from cash register receipts, catalog prices, medical data, or inventory lists. Tasks range from single-step, repetitive actions to very complex responsibilities requiring initiative and planning on the part of the employee. For example:

- A supervisor may present a problem orally to an employee (e.g., "Figure up our total sales for yesterday").

- An employee may perform routine calculations (e.g., figuring shipping charges for mail orders or determining the currency units to use when giving a customer change).

- An employee may be responsible for determining the task, collecting the data, deciding on the appropriate operations, and performing the calculations (e.g., selecting an area to be planted with a certain type of grass seed, taking necessary measurements, determining how to find the amount of seed needed, and performing the necessary calculations).

With any type of presentation, the same problem-solving skills are necessary. Practice, however, should emphasize working with mathematics in simulated workplace situations as well as with written problems. Learners who can choose an appropriate process for a problem presented in this way should have little difficulty with a similar written problem unless they lack the necessary reading skills. If this appears a likely problem, it may be appropriate to also evaluate that learner's reading skills.

To tie instruction to the workplace, have learners keep journals of math problems they actually encounter on the job or at home. Problems that are appropriate for this level can then be shared with the group. Whenever possible, use realistic resources and props such as a group of grocery or other retail items, catalogs, price lists, time cards, or inventories as the basis for problems you present to the learners. You should also give some emphasis to common workplace vocabulary. To read the problems on the *Applied Mathematics* assessment and/or to tackle similar applications on the job, learners need to understand such words as *fare, per, deductions, instructions, commission, discount, markup, budget, contract, overtime,* and *expense*.

PROBLEM-SOLVING STRATEGIES

Whether or not calculators are used in the workplace, learners should still develop some type of problem-solving strategy. Using **estimation** as a tool to predict answers and to check results should be one part of this strategy. By using estimation, learners can determine whether an answer is reasonable compared to something they already know. For example, if a car usually averages 26 miles per gallon of gas, a comparison would indicate an error if computation of that car's mileage rate in a specific instance results in an answer of 260 miles per gallon. Comparisons are also useful in checking measurements. This skill enables individuals on the job to quickly check their own calculations. Estimation can be used at the beginning or at the end of the process, can be accomplished in a number of ways, and can serve at least two purposes.

- Learners may use estimation before solving a problem to help them *determine which operation to use.* Learners should ask themselves if the answer should logically be larger or smaller than the facts that are given. In both Figure 3.1 and Figure 3.2, it is reasonable to think that the customer would owe more than the cost of any one CD if he or she is buying three CDs; thus, the answer would be larger. At Level 3, learners should realize that if they are working with positive numbers larger than 1, a larger answer requires that they should either add or multiply to solve the problem.

♦ By rounding off numbers, learners can determine the *approximate range of the answer.* For example, if the compact discs in Figure 3.2 were rounded off to $15, $13, and $9, the answer would be $37, indicating that the actual answer is close to $37. Using estimation to check calculator results can help learners avoid the results of input errors. Two errors commonly made when using calculators are entering the numbers incorrectly and placing the decimal point incorrectly. Learners should be encouraged to round off numbers to help in their estimation.

Besides estimation, an orderly series of steps is needed to actually solve a problem. At first, learners may need to consciously use and extensively practice such a strategy. But the goal should be to internalize an approach so it can be used quickly and efficiently in the workplace. Regardless of the model used, any successful problem-solving strategy should deal with the following questions:

■ **What is the question?**

■ **What are the facts?**

■ **What steps of reasoning and which process should be followed using these facts to answer this particular question?**

What is the question? In all problem situations, learners should be clearly aware of what they are looking for. It is impossible to determine the important facts or the direction to go without this information.

The problem in Figure 3.1 is typical of Level 3 problems in several ways: only one operation needs to be performed to solve it, there are only whole numbers and decimals, and all of the information necessary to solve the problem is readily available.

> You work in a music shop and a customer wants to purchase 3 compact discs costing $14.99 each. Excluding any taxes, how much does the customer owe?

Figure 3.1

Solution
Step 1: $3 \times \$14.99 = \$44.97$

As practice for the workplace, a problem such as the one above could be acted out. Those learners not participating should take notes and try to determine for themselves what the question is. Have all learners estimate the answer and then discuss any differences. Encourage learners to *restate* the question: What is the total cost of the CDs? This allows you to check the learners' understanding of the question.

Karl J. Smith, in his book *Problem Solving,* recommends a problem-solving approach that revolves around translating the problem into a verbal expression. Specifically, he recommends using descriptive phrases instead of letters for all the variables until the last step.

> Read the problem carefully. Make sure you know what is given and what is wanted. Next, write a verbal description (without using variables), using operation signs and an equal sign, but still using the key words. This is called *translating* the problem.

$$10 + 2 \times (A\ NUMBER) = 22$$

> When there is a single unknown, choose a variable. IMPORTANT: Do not BEGIN by choosing a variable; choose a variable only AFTER the problem has been simplified to a single variable. With more complicated problems, you will not know at the start what the variable should be.[1]

What are the important facts? The restatement activities described above should enable the learner to deal with the second question: What are the important facts? The word *cost* tells the learner to look for a monetary amount (i.e., $14.99), and the words *total* and *CDs* indicate that the number of CDs is also an important fact. At first, learners should note all numbers that appear related to the task and any words that may be clues to the operation. Emphasize, however, that not all of this information may actually be used, and that the context of the word clues should be taken into account when developing a full understanding of the problem. Learners should also practice using their resulting answers to verify their use of the word clues. Encourage discussion of each problem to help learners develop the habit of verification.

What operations should be used on these facts to answer this particular question? Using word clues is crucial to determining the appropriate operation(s). It is probably easier for most learners to look at a textbook word problem and isolate the words that point to the correct operation than it is for them to apply this practice to a workplace situation. Encourage learners at this level to describe the situation, even in writing at first, and watch for words and phrases that are related to the question, the facts, and the operation. Make lists of these words and discuss their choices.

Once learners have determined the question, the facts, and the word clues, a process can be developed for determining the correct operation. Review word clues that indicate certain operations. For example, words like *have left, remain, difference,* and *change* all point to using the operation of subtraction. In cases where the word clues are not obvious, restate the question using the words *in all, left, total,* or *for each* to determine which words best fit the meaning of the question. In Figures 3.1 and 3.2, the question can be restated: How much does the customer owe *in all?* Or: What is the *total* cost? This would indicate that multiplication or addition should be used.

In addition to finding word clues, have learners practice using the **given information** to determine the correct operation. The given information includes the total value in subtraction and division problems. However, in addition and multiplication problems, the total value is always part of the question. Learners should consider what is given in the problem and what must be calculated (see Figure 3.2).

> A customer in the music shop where you work purchases 3 compact discs. One costs $14.99, one costs $12.99, and one is on sale for $8.99. Excluding any taxes, how much does the customer owe?

Figure 3.2

Solution

Step 1: $14.99 + $12.99 + $8.99 = $36.97

This question can appropriately be restated: What is the *sum* of the prices of the CDs? The cost of the individual CDs is given, but the total is unknown. You may want to review the approach that multiplication is merely a shortcut for addition that can be used when all of the numbers being totaled are the same.

Remind learners that problems dealing with unequal units call for addition or subtraction and those with equal units require division or multiplication. Have learners use a table such as the one in Figure 3.3 with some numbers missing for practice in determining whether division or multiplication is needed. The emphasis should be on how the correct operation is chosen.

UNIT	NO. OF UNITS	VALUE	TOTAL VALUE
crate of items	?	12 items per crate	36
purchased item	5	$? per item	$6.25
distance (mile)	7	2 hrs per mile	? hrs

Figure 3.3

POSITIVE AND NEGATIVE NUMBERS

In the workplace, positive and negative numbers are commonly used to denote values or to show the direction of a process, such as stock market prices, bookkeeping amounts, temperature, and map and graph coordinates. A thermometer is perhaps the easiest and most familiar tool to use when working with learners who have difficulty understanding the concept of signed (positive or negative) numbers. Before working with the mathematic processes on signed numbers, you may want to have learners practice writing expressions of positive and negative values from a list of verbal statements (e.g., 7°F below zero, $1.25 price gain, $27.32 overdrawn, 500 feet below sea level, 22°F above zero). Figure 3.4 shows how signed numbers might be used in the workplace.

> You must keep track of inventory in an office supply warehouse. This week, 8 computers of a particular model have been shipped out of the warehouse to a local store, while 4 more computers of the same model have been received by the warehouse from the factory. What is the overall change in the number of these computers in inventory this week?

Figure 3.4

Solution

Step 1: Change = number received – number shipped = 4 – 8 = –4

CHANGING NUMBERS FROM ONE FORM TO ANOTHER

Besides working with basic mathematical operations, learners at Level 3 must be able to change numbers from one form to another. For practice with these conversions, you can devise equivalency charts, like the one shown in Figure 3.5, with two forms of each fractional number left blank. There are actually two goals to keep in mind. First, learners should be familiar enough with these conversions to readily recognize and use common equivalents (e.g., $\frac{1}{2}$ = 50% = 0.5). Second, learners should also become adept at converting the less common numbers from one form to another.

Fraction	Decimal	Percent
$\frac{1}{2}$	?	?
?	0.85	?
?	?	40%

Figure 3.5

OTHER STRATEGIES FOR IMPROVING SKILLS TO LEVEL 3

Level 3 problems usually involve only one step. Some of the situations below can be altered to create several related problems. The icons (first shown on page 2) indicate the application(s) involved.

Using grocery or other ads, or using catalogs, have learners total prices for lists of specific items.

Using the same ads or catalogs, have learners figure the amount of change due a customer when given a five-, ten-, or twenty-dollar bill.

Have learners determine the amount of money that would be raised if each member of a group sells an equal number of tickets or other items.

Devise or obtain a job time sheet that lists the hours worked by several employees in a specific time period. Have learners total the hours for each employee. Use some fractions of hours.

Using airline schedules, have learners figure the amount of time passengers would spend in the air or between flights.

Have learners use road maps to figure total mileage for more than one route between two cities.

Have learners figure the cost for carpet or flooring given square yards and price per square yard.

Give learners drawings of a utility meter at two different points in time and have them figure the usage.

Using catalogs of items such as office supplies, devise exercises involving bulk orders. For instance, if an office will need 600 ballpoint pens in the next year, how many boxes of one dozen each should be ordered? In addition, learners could figure the total cost of the order or the cost of an order of several items.

Level 3 Sample Items

Problem 3.1

In your job as a cashier, a customer gives you a $20 bill to pay for a can of coffee that costs $3.84. How much change should you give back?

    A. $15.26
    B. $16.16
    C. $16.26
    D. $16.84
    E. $17.16

Solution

Step 1: $20.00 – $3.84 = $16.16

Indications that this is a Level 3 problem:

♦ It is a one-step word problem with one subtraction operation. Adding the cost of other items or figuring the tax would move this problem to a Level 4.

♦ The information is in a logical order; that is, it is given in the same order in which the problem would be set up.

♦ There is no extraneous information.

The question can be restated, How much change is *left* out of the $20 bill? Therefore, this is a subtraction problem. The important facts are the total value, $20, and $3.84, the cost of the coffee.

Problem 3.2

It took you 1 hour to unpack, price, and shelve 3 boxes of jeans at work. On the average, how many minutes did it take you to unpack, price, and shelve 1 box of jeans?

    A.   15
    B.   20
    C.   30
    D.   40
    E.   60

Solution

Step 1: 1 hour = 60 minutes;
        60 minutes ÷ 3 boxes = 20 minutes/box

Indications that this is a Level 3 problem:

♦ The problem involves one operation and whole numbers.

♦ There is no extraneous information.

The question can be restated, How long did it take to unpack *each* box of jeans if you did them all at the same rate? Division is the process needed. The problem gives the number and kind of units, *3 boxes of jeans,* and the total value, *1 hour.* The learner is looking for the value, or time per unit. This problem also requires conversion of 1 hour into 60 minutes.

4 Description of Level 4 Skills

At Level 4, tasks may present information out of order and may include extra, unnecessary information. A simple chart, diagram, or graph may be included. In addition to demonstrating the skills at Level 3, individuals with Level 4 skills can:

- **Put the information in the right order before they perform calculations.** The problems at this level must be carefully analyzed to determine what information is needed to solve them and to determine which operation(s) to perform and the order in which to perform them.

- **Solve problems that require one or two operations.** They may add, subtract, or multiply using several positive or negative numbers (such as 10 or –2), and they may divide positive numbers (such as 10).

- **Figure out averages, simple ratios, simple proportions, or rates using whole numbers and decimals.** Averages calculated as $\frac{(10 + 11 + 12)}{3}$, ratios such as $\frac{3}{4}$, and proportions like $\frac{3}{5} = \frac{x}{15}$ are common in the workplace, as are rates such as 10 mph or units per hour.

- **Add commonly known fractions, decimals, or percentages.** Commonly known fractions, decimals, or percentages such as $\frac{1}{2}$, 0.75, or 25% are widely used in the workplace. Individuals at this level are familiar enough with these to make mental conversions.

- **Add three fractions that share a common denominator.** Examples such as $\frac{1}{8} + \frac{3}{8} + \frac{7}{8}$ are often encountered in the workplace.

- **Multiply a mixed number by a whole number or decimal.** Quantities such as $2\frac{1}{8} \times 6.8$ must be converted so that both are either improper fractions or decimals.

Moving to Level 4 Skills

With each introduction of a new skill, you should present as many work-related problems and activities as possible. If these activities illustrate connections with other mathematical skills, as well as with outside experiences that the learner already has, the new material will become much more meaningful. At the same time, stress multiple methods of solution and encourage learners to raise questions, discuss different approaches, and examine the advantages and disadvantages of each method.

Use calculators whenever possible to familiarize learners with their use, limitations, and idiosyncrasies. For example, learners should be aware that on some calculators an attempt to multiply two negative numbers will result in subtraction, not multiplication.

MULTIPLE-STEP PROBLEMS

At Level 4, learners must be able to perform more than one step or mathematical operation. A major difficulty with word problems for many learners, whether in a textbook or in a real-life situation, is determining what operations to use and in what order to use them. Stress the problem-solving strategies for Level 3, which are described on pages 7 and 8 of this *Target.* Consider Figure 4.1:

You work in a bakery. A customer orders a dozen doughnuts priced at $5.75 a dozen and 4 cinnamon rolls at 75¢ a roll. How much should you charge the customer for this order?

Figure 4.1

Solution

Step 1: $4 \times 75¢ = 300¢ = \$3.00$

Step 2: $\$3.00 + \$5.75 = \$8.75$

To help learners solve this problem, either have them role-play the situation or have them visualize two orders: (1) an order for 1 dozen doughnuts and (2) an order for 4 rolls. The important facts are the total cost of the doughnuts and the total cost of the rolls. With this information, learners should recognize that the total cost of the rolls is not given and should be calculated. Because there are 4 rolls at 75¢ **each,** multiplication is the operation indicated. Learners can then add the two separate costs to arrive at what the customer owes. Stress that a multiple-step problem is really just two or more separate problems related in some way. Have learners determine what the separate problems are in multiple-step problems and in what order the calculations should be performed.

AVERAGES

At Level 4, learners are required to average decimals but not mixed numbers. An important guideline for learners to remember when estimating answers is that an average is never smaller than the lowest number or larger than the highest number.

> As the sales manager of an automobile dealership, you are expected to figure the average number of sales per month at the end of each quarter. In the first quarter of this year, 17 cars were sold in January, 25 in February, and 15 in March. What was the average number of sales per month of the first quarter?

Figure 4.2

Solution

Step 1: Total = 17 + 25 + 15 = 57

Step 2: Average = 57 ÷ 3 = 19

Problems relating to finding averages are plentiful in the workplace. Averages of weather-related information, such as rainfall, temperature, and snowfall, are important to many occupations. Attendance at various events, monthly or yearly sales, and the weight gain or loss of a specific group of hospital patients are other examples. Learners who have trouble with the concept of averages may find it helpful to work with diagrams or objects that can be used to model the problem.

RATIOS AND PROPORTIONS

The ratios and proportions introduced at Level 4 use whole numbers only. A ratio expresses the relationship of one quantity to another, and proportions contain equivalent ratios in which one of the terms may be a variable. Understanding how quantities in a proportion relate to each other is useful in solving such workplace problems as finding unit cost, rates of use or production, and scale measurements. Using area models to illustrate the ratios in a proportion also allows learners to visually compare the ratios. Learners need to remember that the order of the items in a ratio pair is extremely important. Consider Figure 4.3:

> In the hospital where you work, one of your duties is to take pulse counts. One patient has a pulse count of 21 beats in 15 seconds. At this rate, what should this patient's pulse count be for 60 seconds?

Figure 4.3

Solution

Step 1: Set up proportion:

$$\frac{21 \text{ beats}}{15 \text{ seconds}} = \frac{\text{how many beats}}{60 \text{ seconds}}$$

Step 2: Rearrange and solve:

$$\frac{\text{how many beats}}{1} = \frac{21 \text{ beats}}{15 \text{ seconds}} \times 60 \text{ seconds} = 84 \text{ beats}$$

An individual at this level may work the problem by reasoning that a rate of 21 beats in 15 seconds would result in 42 beats in 30 seconds, 63 beats in 45 seconds and 84 beats in 60 seconds. Dividing and multiplying in the wrong order could have serious results in this instance. The following are examples of other proportion problems that you can adapt for practice.

Mixtures: You use 3 gallons of concentrated cleaner to mix 10 gallons of usable solution. How much concentrated cleaner should be used to make 30 gallons of usable solution?

Solution

Step 1: $\dfrac{3 \text{ gallons}}{10 \text{ gallons}} = \dfrac{\text{how much cleaner}}{30 \text{ gallons}}$

Step 2:

$\dfrac{\text{how much cleaner}}{1} = \dfrac{3 \text{ gallons}}{10 \text{ gallons}} \times 30 \text{ gallons} = 9 \text{ gallons}$

Materials: If it takes 2 yards of interfacing to make 4 suits, how many yards of interfacing should it take to make 20 suits?

Solution

Step 1: $\dfrac{2 \text{ yards}}{4 \text{ suits}} = \dfrac{\text{how many yards}}{20 \text{ suits}}$

Step 2:

$\dfrac{\text{how many yards}}{1} = \dfrac{2 \text{ yards}}{4 \text{ suits}} \times 20 \text{ suits} = 10 \text{ yards}$

Production Rates: If you can print 800 copies of a letter in 16 minutes, how long should it take to print 2400 copies of the same letter?

Solution

Step 1: $\dfrac{800 \text{ copies}}{16 \text{ minutes}} = \dfrac{2400 \text{ copies}}{\text{how many minutes}}$

Step 2: $\dfrac{16 \text{ minutes}}{800 \text{ copies}} = \dfrac{\text{how many minutes}}{2400 \text{ copies}}$

Step 3:

$\dfrac{\text{how many minutes}}{1} = \dfrac{16 \text{ minutes}}{800 \text{ copies}} \times 2400 \text{ copies} = 48 \text{ minutes}$

PERCENTAGES

At Level 4, learners must be able to find percentages of monetary amounts, such as taxes, commissions, and discounts. These problems involve straightforward calculations with both terms in whole numbers. Learners need to remember that when working with percentages and numbers in different forms within the same problem, they usually should convert the percentage either to a decimal number by moving the decimal point two places to the left, or to a fraction by putting the percentage over 100. When using a calculator, the decimal method is usually the easier (see Figure 4.4).

A customer purchased clothing totaling $54.00 from the clothing store where you work. The state sales tax rate is 5%. How much tax should you charge the customer for this purchase?

Figure 4.4

Solution

Step 1: 5% = 0.05

Step 2: $54.00 × 0.05 = $2.70

FRACTIONS

Level 4 introduces two types of problems involving the addition of fractions. A problem may require the addition of three fractions that share a common denominator or the multiplication of a common fraction by a whole number. Some learners have difficulty with the concept that multiplication can result in a number that is smaller than the whole number in the problem. It may be helpful to point out that in a problem such as $\frac{3}{4} \times 8$, the times symbol represents the word *of*. The learner should be trying to find $\frac{3}{4}$ of 8 and should realize that if the fraction in this type of problem is less than 1, the answer should be less than the whole number.

DIAGRAMS AND GRAPHS

At Level 4, learners must be able to use simple diagrams or graphs to obtain information to solve problems. Using a graph like the one shown in Figure 4.5, have your learners point out the parts, including the horizontal and vertical axes, location of headings, and placement of data.
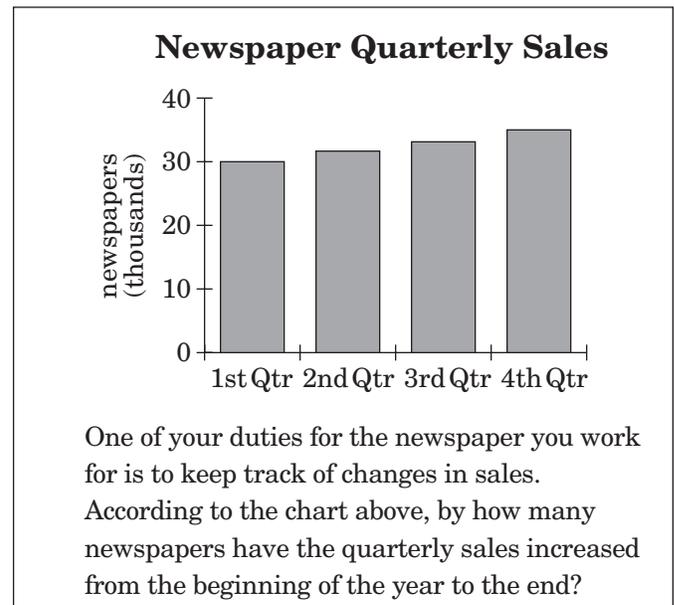
Newspaper Quarterly Sales

One of your duties for the newspaper you work for is to keep track of changes in sales. According to the chart above, by how many newspapers have the quarterly sales increased from the beginning of the year to the end?

Figure 4.5

Solution

Step 1: Read the bar heights for 1st and 4th quarters.
1st quarter = 30 thousand
4th quarter = 35 thousand

Step 2: Find the increase.
4th quarter – 1st quarter =
35 thousand – 30 thousand =
5 thousand = 5,000

Learners should notice first that each bar represents one quarter of the year, designated along the horizontal axis. The vertical axis shows the newspaper sales in thousands. In order to work the problem in Figure 4.5, learners must be able to specify what numbers the bars for the first and fourth quarters represent. You could also ask learners to use this graph to determine the average quarterly sales or to find the total sales for the year.

OTHER STRATEGIES FOR IMPROVING SKILLS TO LEVEL 4

Using prices from catalogs or ads, have learners figure totals and then a percent tax or discount. Filling out order blanks with several items, shipping charges, and tax is also a good method for practicing multiple-step problems.

Have learners figure averages based on graphs and diagrams from newspapers.

For practice with fractions, have learners double recipes or other mixtures using fractional amounts.

Working with the stock market page, have learners simulate stock purchases by starting with a set amount (e.g., $1,000) and following their stocks, keeping track of price changes.

Using checkbooks with several entries, have learners figure the balance.

Give learners the data on miles driven and gas used for each of several cars or trucks and have them figure the miles per gallon for each. Once they have done this, they can also calculate the average rate for the fleet. Learners could also be given the mileage rate and then asked to calculate the amount of gas needed for a 1,500-mile trip.

Have learners use proportions to calculate mixture amounts from the instructions on containers of lawn fertilizer, caulking compound, or other similar products.

Using distance and average speed, have learners calculate the time required for a specific trip.

Have learners use proportions to make scale drawings of the classroom or parking lot, stressing the accuracy of the scale.

Level 4 Sample Items

Problem 4.1

Over the last five days, you made the following number of sales calls: 8, 7, 9, 5, and 7. To help in planning staff time, you track the average number of calls you make each day. What was your average over the last 5 days?

- **A.** 4.1
- **B.** 7.0
- **C.** 7.2
- **D.** 9.0
- **E.** 36.0

Solution

Step 1: Total: 8 + 7 + 9 + 5 + 7 = 36

Step 2: Average: 36 calls ÷ 5 days = 7.2 calls/day

Indications that this is a Level 4 problem:

- ♦ The problem involves two operations on several positive numbers.
- ♦ The problem involves finding an average.

This problem is very direct and does not contain extraneous information. Learners should use estimation to determine that the answer will be greater than 5.0 and less than 9.0.

Problem 4.2

The discount store where you work is selling a video game for 15% off the regular price of $39.00. You have to change the price tags for the sale. How much should you take off from the regular price?

    A. $0.15
    B. $1.50
    C. $2.60
    D. $5.85
    E. $7.00

Solution

Step 1: 15% = 0.15

Step 2: $0.15 \times \$39.00 = \$5.85$

Indications that this is a Level 4 problem:

- The problem involves one straightforward operation on simple decimal numbers.

- The problem involves finding a percent of a monetary amount.

It is especially important for learners to understand exactly what the question is asking: How much is the discount? The 15% discount can be found by converting 15% to 0.15, which is then multiplied by the regular price of $39.00.

Problem 4.3

You need about $1\frac{1}{2}$ hours to set up a computer workstation. At this rate, how many hours should it take you to set up 7 of these workstations?

    A. $4\frac{2}{3}$

    B. $8\frac{1}{2}$

    C.   10

    D. $10\frac{1}{2}$

    E. $11\frac{2}{3}$

Solution

Step 1: Set up ratio and proportion as

$$\frac{1\frac{1}{2}\text{ hours}}{1\text{ computer station}} = \frac{\text{how many hours}}{7\text{ computer stations}}$$

Step 2: Rearrange:

$$\frac{\text{how many hours}}{1} = \frac{1\frac{1}{2}\text{ hours}}{1\text{ computer station}} \times 7\text{ computer stations}$$

Step 3: $1\frac{1}{2} \times 7 = 10\frac{1}{2}$ hours

As an alternative, this problem can be looked at as a rate problem.

Step 1: Recognize the rate as 1 station per $1\frac{1}{2}$ hours.

Step 2: $7 \times 1\frac{1}{2} = 10\frac{1}{2}$ hours

Indications that this is a Level 4 problem:

- One mathematical operation (i.e., multiplication) is required.

- The problem involves multiplication of a mixed number by a whole number.

This problem is an example of how a proportion often provides a problem-solving approach. The learner who understands that $1\frac{1}{2}$ hours : 1 computer station = ? hours : 7 computer stations has a clear grasp of the problem's structure.

5 Description of Level 5 Skills

Level 5 tasks present several steps of logic and calculation. For example, at this level individuals may complete an order form by totaling an order and then computing tax. In addition to demonstrating the skills at the previous levels, individuals with Level 5 skills can:

- **Decide what information, calculations, or unit conversions to use to find the answer to a problem.** Many workplace problems present more information than is necessary to solve them. Individuals with Level 5 skills are able to eliminate the extra information and choose the necessary data.

- **Calculate perimeters and areas of basic shapes.** Individuals with Level 5 skills are able to recognize the correct formula and make accurate substitutions of data in the formula in order to find the perimeter or area of rectangles, triangles, and circles.

- **Look up a formula and change from one unit to another in a single step within a system of measurement or between systems of measurement.** Individuals at this level use available formulas for conversions such as ounces to pounds or pounds to kilograms.

- **Calculate using mixed units.** Individuals are adept at converting units of measurement in order to work with numbers such as 3.50 hours or 4 hours 30 minutes.

- **Divide negative numbers.** At this level, division involving negative quantities should pose no problem.

- **Calculate percent discounts or markups.** At this level, individuals work with multiple-step calculations and work with percentages to calculate discounts and markups.

- **Identify the best deal by doing one- and two-step calculations.** Individuals at Level 5 can select the appropriate information to make two separate calculations and then compare to determine the solution that meets the stated conditions.

Moving to Level 5 Skills

At Level 5, the pertinent information and appropriate operations needed to solve the problems are less obvious than at the lower levels. At Levels 3 and 4, the tasks are clear and the facts obvious without much clutter in the form of extraneous information (e.g., totaling a list of numbers, making change, figuring sales tax). At Level 5, some extraneous information is present. Regardless, instruction can focus on the same approach discussed at the previous levels (i.e., identify the question, find the important facts, and determine the appropriate operation).

EXTRANEOUS INFORMATION

Problems from the workplace already containing extraneous information, as many do, will be much more effective for instructional purposes than simple problems that have had extraneous information artificially added. In workplace tasks, individuals often confront more information than is actually needed to solve a particular problem. Price tags may include an original price as well as a discount price, for example. A customer at a paint store may bring in more measurements than are needed to figure the amount of paint required. You may want to have learners list, highlight, or note in some other way the necessary information after they have determined a process to solve the problem at hand.

PERIMETER AND AREA

In the workplace, employees are often presented with measurement problems that necessitate finding area or perimeter, even though these terms may not actually be used in instructions to employees. For example, an employee may be asked to measure a room for carpet rather than being given the specific instruction to find the area of the floor. Or the employee may be asked to determine how much fencing is required to surround a property, instead of the perimeter of the piece of land. It is important for learners to practice on tasks presented in this way.

To deal with such tasks, learners should practice restating the question and then determining a way to solve the problem. The obvious method to find area or perimeter is to use the appropriate formula, but learners can benefit from attempting to develop their own methods. This can also help them to understand and remember the formulas. Except for time and U.S. currency, formulas for conversions necessary to work the problems in the *Applied Mathematics* assessment are provided (see Appendix A), since employees would have access to such aids in most workplaces. In addition, learners should be familiar with terms such as *area* and *perimeter* and should be able to determine which formula is appropriate for the problem.

You may first want to determine if your learners are able to distinguish between perimeter and area and if they are able to apply the terms in specific situations. If problems exist with understanding the definitions of perimeter and area, some visual illustration may help. Use two figures that have the same perimeter but different areas to illustrate the difference, such as those in Figure 5.1.
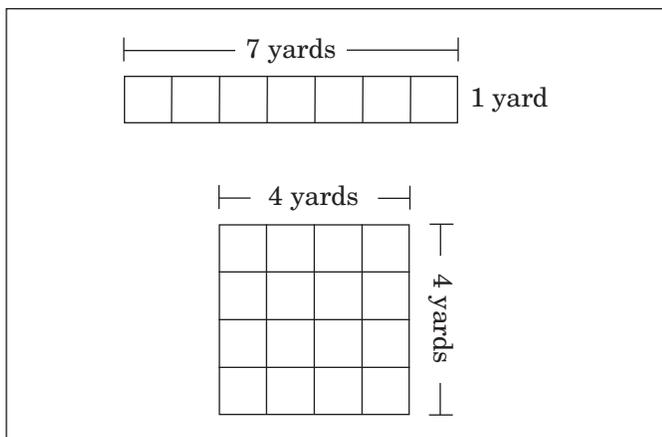

Figure 5.1

Learners should see that while both models have a perimeter of 16 yards, the model on the bottom has an area of 16 square yards and the one on the top has an area of only 7 square yards. Again, word clues can be very helpful. Words denoting linear distance around a boundary indicate the need to use a formula for perimeter or circumference. Have learners use their answers to verify the process they followed. They can use models like those in Figure 5.1 to verify results or to develop their own formulas through trial and error.

The models shown in Figure 5.1 can also be used to illustrate the idea of area. Learners can use the length and width measurements to find the area and then check their results by counting the actual squares. Depending on the models you use, you may need to caution your learners to convert both measurements to the same unit before multiplying.

Learners should understand that references to measurements such as square feet, inches, and miles probably indicate the use of a formula (see Figure 5.2). Besides being able to choose the correct formula, the learner must also be able to provide the correct substitution in the formula. For example, while the base and height measurements are interchangeable in the formula for the area of a triangle, learners should understand that the height must be the perpendicular measurement drawn from the side used as the base to the opposite angle. Also, because many learners confuse radius and diameter when dealing with circles, you may need to stress the difference between these terms. One simple memory trick is to remember that radius is the shorter word and the shorter measurement.

You are a new maintenance person for the local school and need to calculate how much wax will be required for the cafeteria floor based on its square footage. The cafeteria is 120 feet by 45 feet. What is the square footage you should use to figure the amount of wax needed for the cafeteria floor?

Figure 5.2

Solution

Step 1: Area = length × width =
120 feet × 45 feet = 5,400 square feet

DENOMINATE NUMBERS

Other measurement problems in the workplace involve **denominate** numbers; that is, numbers used to quantify a unit of measurement. Denominate numbers are used to express measurements of temperature, weight, time, liquid, volume, angularity, distance, and area, among others. Currency amounts are also denominate numbers.

Difficulties arise for many learners when they encounter **compound denominate** numbers, which are measurements that include two or more units of measure (e.g., 4 hours 5 minutes, 1 gallon 3 pints, 3 feet 8 inches). In order to work with compound denominate numbers, learners must be able to convert from one unit to another and they must understand how these conversions affect the operations required to solve a problem. In the *Applied Mathematics* assessment, the conversion tables provided include standard conversions of distance, area, volume, weight, temperature, and electricity. The examinee is expected to know standard time and U.S. currency conversions without the conversion formulas.

Mathematical processes can be done on compound denominate numbers using two different methods. In the first, the fundamental process of carrying or regrouping can be used for addition and multiplication problems in the same manner as when working with non-denominate numbers. In subtraction problems, the basic method of exchange or borrowing is used, except that, again, the exchange must fit the measure.

The other method of dealing with all four fundamental operations on denominate numbers is to convert all the units to the smallest unit, perform the appropriate operation, and then convert back to the largest unit or compound form possible. Have learners practice both methods, emphasizing the use of a calculator. Examples of workplace tasks involving work with compound denominate numbers are plentiful and easy to devise, such as adding several different weights together, dividing time into parts, and subtracting one volume from another.

FRACTIONS

Fractions appear most often in tasks dealing with quantity, averages, and measurement. Fractions at Level 5 may have exponents or several terms in the numerator or denominator. Also, tasks may involve addition and subtraction of fractions with unlike denominators, multiplication and division of fractions with like denominators, or multiplication of a fraction by a decimal.

You should review the simplification of fractions. You can set up the problem in Figure 5.3 using a complex fraction that can then be simplified:

You must set up tables for a wedding reception in the restaurant where you work. There will be 24 individuals and 40 couples attending. Each table seats 8 persons. How many tables should you set up?

Figure 5.3

Solution

Step 1: Calculate the number of tables:

$$\frac{40 \text{ couples} \times \frac{2 \text{ individuals}}{\text{couple}} + 24 \text{ individuals}}{\frac{8 \text{ individuals}}{\text{table}}} =$$

$$\frac{104 \text{ individuals}}{\frac{8 \text{ individuals}}{\text{table}}} = 13 \text{ tables}$$

Once the problem has been set up, learners may need to review the order of operations and the functions of parentheses.

Exponents are encountered at this level, also. Learners need to understand that exponents at this level are usually indicators of how many times the base number is used as a factor (e.g., $5^2 = 5 \times 5$). The most common mistake with exponents is to use the exponent as a factor (e.g., $5^2 = 5 \times 2$). Exponents may appear in equations and/or in fractions.

The addition and subtraction of fractions with unlike denominators at Level 5 may call for review or instruction in finding a common denominator. Learners may simply multiply one denominator by the other and use the result as the common denominator. The result may not be the **least** common denominator, but if the operations are being performed on a calculator, using larger numbers does not involve any extra time or effort and may help to avoid errors.

Level 5 also includes multiplication and division of fractions with like denominators. You will probably want to deal with these two operations together, since the division of fractions is usually defined in terms of multiplication.

If a problem calls for multiplying a fraction by a number with a decimal, learners can either convert both numbers to fractions or both to decimals. This type of problem could occur in the situation found in Figure 5.4:

> The shoe store where you work is having a $\frac{1}{4}$ off sale. A customer purchases a pair of shoes that regularly costs $48.50. How much should you deduct from the regular price?

Figure 5.4

Solution

Step 1: $\frac{1}{4} = 1 \div 4 = 0.25$

Step 2: $0.25 \times \$48.50 = \$12.125$ rounded to $12.13

When using common fractions and a calculator, learners may want to convert the fraction to a decimal and then multiply that number by the regular price (e.g., 0.25×48.50). If learners do not know the decimal equivalent of the fraction, they should be able to find it through division.

PERCENTAGE

Percentage problems at Level 5 include finding what percent one number is of another (see Figure 5.5) and calculating a percent of a fraction or a decimal. Have learners practice dividing the rate by the base to find what percent one number (the rate) is of another (the base). Remind learners that the word *of* is usually a clue as to which number is the base, but they should verify for themselves whether this interpretation makes sense.

> As a teacher's aide, you must figure percentage scores on student tests. If a student has 72 points out of 87 possible points, what percent of the total possible points did this student obtain?

Figure 5.5

Solution

Step 1: $72 \div 87 = 0.83 = 83\%$

Discounts and markups are often figured on a percentage basis. For practice, you could ask learners what percent a discount or markup is of the original price. Another appropriate exercise would be calculating the percent increase in sales, production, or employment, or other applications involving money or quantity.

PRODUCTION RATES

Learners should be able to predict rates through the use of proportions, which were introduced at Level 4. Level 5 rate problems involve whole numbers only and often deal with production (see Figure 5.6), which is a combination of quantity and time applications.

> You work in the gift wrap section of a department store. You can wrap 15 average-sized packages in 2 hours. About how many of these packages should you be able to wrap in 5 hours?

Figure 5.6

Solution

Step 1: $5 \text{ hours} \times \dfrac{15 \text{ packages}}{2 \text{ hours}} = 37.5$ rounded to 37

Besides rates of production, Level 5 employees may also need to be able to predict rates of use, rates of growth, and rates of speed, among others. For example, knowing how much of a material or product is used in one month allows one to decide when that material should be reordered.

BEST DEAL

Best deal problems, usually involving several calculations, are introduced at this level. A clue to best deal problems is usually the use of words such as *lower cost, cheaper, save, less expensive, minimum,* and other comparative terms. However, learners at this level will have a better grasp of these clues if they develop their own lists from groups of best deal problems. In this type of problem, learners should understand that unless they are told one of the prices or costs, they may have to make at least two separate calculations in order to make a comparison. Thus, one important consideration is separating the data for each potential service or product. Consider Figure 5.7:

> Quick Call charges 18¢ per minute for long-distance calls. Econo Phone totals your phone usage each month and rounds the number of minutes up to the nearest 15 minutes. It then charges $7.90 per hour of phone usage, dividing this charge into 15-minute segments if you used less than a full hour. If your office makes 5 hours 3 minutes worth of calls this month using the company with the lower price, how much will these calls cost?

Figure 5.7

Solution

Step 1: Change time of calls to minutes:

$5 \text{ hours} \times \dfrac{60 \text{ minutes}}{\text{hour}} + 3 \text{ minutes} = 303 \text{ minutes}$

Step 2: Quick Call charges:

$303 \text{ minutes} \times \dfrac{18¢}{\text{minute}} = 5454¢ = \$54.54$

Step 3: Change extra minutes to hours, rounding up to the nearest 15-minute interval:

$3 \text{ minutes} \div \dfrac{60 \text{ minutes}}{1 \text{ hour}} = 0.05$ rounded up to 0.25 hours

Step 4: Econo Phone charges:

$5.25 \text{ hours} \times \$7.90/\text{hour} = \$41.48$

Step 5: Select the lower charge: $41.48 is the lower.

In order to decide which company has the lower cost for this particular office, learners should first calculate the charges at each rate. The information may be clarified by making a simple chart detailing the rates.

OTHER STRATEGIES FOR IMPROVING SKILLS TO LEVEL 5

Have learners reduce lists of mixed denominate numbers, such as 10 hr 90 min, 4 yd 2 ft 15 in, and 3 gal 6 qt 5 pt.

Practice best deal skills using materials found in the consumer market. Telephone services, photography studios, day care centers, car rental agencies, and many other services offer varied pricing techniques. In order to make accurate comparisons, you will need to identify specific situations in which these services will be used.

Use prices of grocery items to figure unit prices.

Use charts of shipping costs to figure charges on weights that include fractions, decimals, or units that must be converted.

Have learners calculate the appropriate quantities of ingredients for certain amounts of solutions, such as fertilizers, where the directions are given in proportion form (e.g., mix 4:1).

Have learners determine the smallest and largest areas that can be enclosed by a fixed length of fence.

Develop problems finding perimeter and area in order to determine the amounts of building materials needed for a project (e.g., flooring, paint, wallpaper, and decking, or the amount of fertilizer needed for a lawn).

Assign learners the task of increasing a recipe for a large group. Be sure the original recipe includes some fractional measurements. Learners should also practice reducing the resulting compound denominate numbers.

Have learners determine percentage increase or decrease in sales, prices, workforce, or other quantities.

Level 5 Sample Items

Problem 5.1

You have been asked to ship a package that weighs 41 pounds; however, the freight company wants to know the weight in kilograms. What is the closest approximation of the package's weight in kilograms?

A.   18.6
B.   43.2
C.   82.0
D.   90.2
E.   100.0

Solution

Step 1: 41 pounds ÷ 2.2 pounds/kilogram = 18.64 kilograms

The alternate solution:

Step 1: 41 pounds × 453.592 grams/pound = 18,597 grams

Step 2: 18,597 grams ÷ 1,000 grams/kilogram = 18,597 kilograms rounded to 18.6 kilograms

Indications that this is a Level 5 problem:

♦   The problem requires converting between systems of measurement.

If they do not know the conversions, learners can use the conversion table to find that 1 kilogram is approximately 2.2 pounds or that 1 pound equals 453.592 grams. In order to determine the correct procedure to solve the problem, a proportion could be used, or the correct answer could be found by dividing 41 by 2.2. Multiplying by 453.592 will also result in the correct answer, but this method requires an extra step: learners should convert grams to kilograms by dividing the answer by 1,000.

Problem 5.2

The drugstore where you work marks up the price of batteries by 35%. You must price some batteries that cost the store $4.80 per package. What price should you put on each package?

A. $1.68
B. $3.50
C. $5.15
D. $6.48
E. $7.38

Solution

Step 1: Understand that 35% = 0.35

Step 2: Markup: $4.80 × 0.35 = $1.68

Step 3: Price: $4.80 + $1.68 = $6.48

Indications that this is a Level 5 problem:

♦ The problem involves more than one step.

♦ The problem requires finding a percentage of a decimal.

Learners should particularly notice the words "marks up" in this problem. The $4.80 price is the base, and 35% is going to be added to that amount rather than subtracted as it would be in a discount problem.

Problem 5.3

In your job at the kennel, you groom dogs. It takes you 1 hour 15 minutes to groom an average-sized dog. Large dogs, however, take 1 hour 45 minutes to groom. Today you have to groom 3 average-sized dogs and 2 large dogs. How much time should it take you to groom all 5 dogs?

A. 3 hours
B. 3 hours 45 minutes
C. 6 hours 15 minutes
D. 7 hours 15 minutes
E. 7 hours 45 minutes

Solution

Step 1: 1 hour 15 minutes × 3 = 3 hours 45 minutes

Step 2: 1 hour 45 minutes × 2 = 2 hours 90 minutes = 3 hours 30 minutes

Step 3: Total time = 3 hours 45 minutes + 3 hours 30 minutes = 6 hours 75 minutes = 7 hours 15 minutes

Indications that this is a Level 5 problem:

♦ The problem involves several steps.

♦ The problem requires calculations using mixed units.

6 Description of Level 6 Skills

Level 6 tasks may require considerable translation from verbal form to mathematical expression. They generally require considerable setup and involve multiple-step calculations. In addition to demonstrating the skills at the previous levels, individuals with Level 6 skills can:

■ **Use fractions, negative numbers, ratios, percentages, or mixed numbers.** Individuals with Level 6 skills are able both to multiply and divide fractions with unlike denominators and to find reverse percentages.

■ **Rearrange a formula before solving a problem.** An example might be $8x = 20 \Rightarrow x = \frac{20}{8}$.

■ **Calculate multiple rates.** In a workplace where Level 6 skills are required, the multiple rates are often in the form of production rates or pricing schemes. Conversions and other calculations are sometimes necessary.

■ **Look up and use two formulas to change from one unit to another unit within the same system of measurement.** At Level 6, individuals are able to convert from ounces to quarts or vice versa by setting up ratios from the equalities 1 cup = 8 fluid ounces and 1 quart = 4 cups.

■ **Look up and use two formulas to change from one unit in one system of measurement to a unit in another system of measurement.** In this case the equalities could include 1 mile = 1.61 kilometers or 1 liter = 0.264 gallons.

■ **Find the area of basic shapes (rectangles and circles).** Individuals at Level 6 are able to rearrange the formula, convert units of measurement in the calculations, or use the result in further calculations.

■ **Find the volume of rectangular solids.** Individuals at Level 6 are able to convert units of measurement so that all are the same before calculating the volume.

■ **Find the best deal and use the result for another calculation.** At Level 6, individuals may deal with added steps in best deal problems.

■ **Find mistakes in Levels 3, 4, and 5 problems.** An individual with Level 6 skills is able to determine if a Level 5 problem has been done correctly and, if not, where the mistake was made.

Moving to Level 6 Skills

At Level 6, the tasks are more complex, both in the number of calculations required and in the wording and organization of the problems. Frequently, several calculations must be performed and the resulting values then compared, converted, and/or used in further calculations. Skills demonstrated at the lower levels (e.g., working with signed numbers, fractions, denominate numbers, averages) are involved in performing more sophisticated tasks. When a formula is required, it may be necessary to transpose it or to convert some of the units before calculations can be made.

Being able to set up a problem in an equation form becomes critical at this level. Problem-solving processes can be compared to using some type of system to organize a file drawer—you can still locate things without a system, but it takes longer and you are more likely to miss something. However, this does not mean that each person's organizational method must be the same.

Learners can benefit from exploring other problem-solving strategies. Many individuals develop their own methods that may be unorthodox but still work. Drawing a picture to illustrate the conditions, the end result, or a comparable problem helps some individuals to work out an approach. Karl J. Smith suggests a number of other approaches, including looking for a similar problem, dividing a problem into simpler parts, and "guess and test."[2] Many people unconsciously use this last approach of making an estimate and then checking to see if it works.

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[2]Karl J. Smith, *Problem Solving* (Pacific Grove, CA: Brookes/Cole Publishing Company, 1991), 1.

Have learners examine and verbalize their own methods for problem solving by having an extensive debriefing session after working a complex problem. Encourage individuals to trace their own solving method and share this with the rest of the group. Learners can then experiment with these methods on other problems. Compare the effectiveness of different methods on a variety of problem types.

MULTIPLE-STEP PROBLEMS

When faced with the problem in Figure 6.1, learners should notice that the question calls for the total yards of fabric, while other measurements are given in feet and inches, indicating that some conversions are necessary. Learners should next consider what calculation(s) should be performed. Finally, the total amount should be converted to yards.

> You must figure the amount of fabric necessary to make a drape for an office window. The finished drape must be $7\frac{1}{2}$ feet long and 5 feet wide. The fabric is wide enough to allow for side hems, but you must determine the length needed. You must allow 8 inches extra at the top and at the bottom to fold under for the hems. How many linear yards of fabric should you need to make the drape?

Figure 6.1

Solution

Step 1: $7\frac{1}{2}$ feet × 12 inches/foot = 90 inches

Step 2: 90 inches + 8 inches + 8 inches = 106 inches

Step 3: 106 inches ÷ 12 inches/foot ÷ 3 feet/yard = 2.94 yards

FRACTIONS

Multiplication and division of fractions with unlike denominators are Level 6 skills and often appear in measurement tasks in the workplace. Although unlike denominators may cause some learners to perceive these problems as more difficult, point out that the process is exactly the same as is used on fractions with like denominators.

PERCENTAGES

Percentage problems at Level 6 may involve **reverse percentages.** These are problems in which learners are given a value that is a specified percentage of an unknown value, and then are asked to calculate the unknown value. For example, a salesclerk who knows the sale price of an item and that the sale price is 25% off the original price may be asked to figure the original price.

RATE PROBLEMS

At Level 6, learners must be able to calculate a time interval and then use that interval in a rate problem (see Figure 6.2).

> As a cosmetologist, you must schedule your own appointments, so you need to know about how long one haircut takes. Today you gave 7 haircuts between 1:45 and 4:15 and had no breaks. Approximately how long did it take you to give each haircut?

Figure 6.2

Solution

Step 1: 4:15 − 1:45 = 3:75 − 1:45 = 2 hours 30 minutes = 150 minutes

Step 2: 150 minutes ÷ 7 = 21.4 minutes

In Figure 6.2, the time interval can first be found, and to do so, the quantity should be converted from hours to minutes. Conversely, learners may be told the rate and have to multiply it by the time interval. For example:

> The production line you work on can assemble amplifiers at the rate of 5 every 30 minutes. At this rate, how long should it take the line to assemble 125 of the amplifiers?

Figure 6.3

Solution

Step 1: 30 minutes ÷ 5 amplifiers = 6 minutes/amplifier

Step 2: 6 minutes/amplifier × 125 amplifiers = 750 minutes

Step 3: 750 minutes ÷ 60 minutes/hour = 12.5 hours or 12 hours 30 minutes

Alternate solution

Step1: 5 amplifiers ÷ 30 minutes × 60 minutes/hour = 10 amplifiers/hour

Step 2: 125 amplifiers ÷ 10 amplifiers/hour = 12.5 hours or 12 hours 30 minutes

There are two ways to set up this rate problem: the rate can be amplifiers/time or time/amplifier. In addition, the learners can use either hours or minutes as the unit of time; they can calculate how many minutes it takes to assemble *each* amplifier and multiply that time by the total number of amplifiers needed. Some rate problems involve figuring more than one rate and making a comparison, as in Figure 6.4.

> In the machine shop where you work, you are required to keep production records on each part for each day. On Wednesday, 45 pieces of a particular part were machined in 3 hours. On Thursday, 52 pieces of the same part were machined in 3.5 hours. On Friday, 112 pieces of the same part were machined in 7 hours. On which day did the shop have the best production rate (the most pieces machined per hour)?

Figure 6.4

Solution

Step 1: 45 pieces ÷ 3 hours = 15 pieces/hour on Wednesday

Step 2: 52 pieces ÷ 3.5 hours = 14.86 pieces/hour on Thursday

Step 3: 112 pieces ÷ 7 hours = 16 pieces/hour on Friday

Step 4: Friday's rate is greater than the other two.

AREA

Formulas used to solve Level 6 problems may require rearrangement. In Figure 6.5, the area and width of a rectangle are given; it is necessary to find the length. Therefore, the formula can be transposed to $l = \frac{A}{w}$.

The firm you work for has contracted to put a new wood floor in a school gymnasium. The floor has an area of 5,000 square feet and is 50 feet wide. Having received only a partial shipment of the flooring, you have enough to cover a total of 3,000 square feet. If you cover the entire width of the floor, how much of the length in feet should you be able to complete at this time?

Figure 6.5

Solution

Step 1: Since $A = l \times w$, $l = \frac{A}{w}$

Step 2: $l = \frac{A}{w} = \frac{3{,}000 \text{ square feet}}{50 \text{ feet}} = 60$ feet

With this problem, it is important for learners to understand that the area to be covered is 3,000 square feet, not 5,000.

Although problems dealing with area are still fairly simple at this level, they may require using the area in further calculations. These types of calculations are illustrated in Figure 6.6:

You are preparing to tile the floor of a rectangular room that is $15\frac{1}{2}$ feet by $18\frac{1}{2}$ feet in size. The tiles you plan to use are square, measuring 12 inches on each side, and are sold in boxes that contain enough tiles to cover 25 square feet. How many boxes of tiles should you order to complete the job?

Figure 6.6

Solution

Step 1: $15\frac{1}{2}$ feet $\times 18\frac{1}{2}$ feet = 286.75 square feet

Step 2: 286.75 square feet ÷ 25 square feet/box = 11.47 boxes

Step 3: Round up to 12 boxes since 11 boxes would not be enough.

The area in this problem is of a simple rectangle, and it can easily be determined by using a calculator and converting the fractions to decimals. However, the tile dimension, which is given in inches rather than feet, could cause confusion. The important facts are that a box of tiles covers 25 square feet and that the tiles are sold by the box only. Normally, the actual answer of 11.47 would be rounded down to 11. But in this case, 11 boxes would not be enough to cover the entire floor; 12 would be needed. It would be useful here to discuss the importance of the degree of accuracy and how it applies to units such as these boxes of tile.

VOLUME

Problems at this level also include figuring the volume of a rectangular solid. Review word clues such as *cubic, capacity,* and *solid* or volume units, such as *gallons,* that indicate volume rather than area is being sought. You should stress that volume is a solid measurement, not just the surface area of a three-dimensional object. The fact, for instance, that 1 cubic foot equals 1,728 cubic inches may seem nearly impossible to some learners. Demonstration of volume compared to area with wooden blocks or similar items may help.

BEST DEAL

Best deal problems at Level 6 include using the results in further calculations. Rather than just comparing two costs and choosing the better, it may be necessary to figure the difference as well. Learners may need practice in breaking these tasks into three separate problems. Using a simple chart at first may be helpful. Take, for example, Figure 6.7:

> The 55 employees in the plant where you work have lunch delivered to the vending area once a week. The charge to individuals is set just high enough to cover the total cost. The current caterer, Yummy Foods, has a $25 delivery fee in addition to the $3.50 charge per person. You have found that Greedy Gourmet has a $40 delivery fee but only charges $3.00 per person. How much can you save each employee on the cost of a meal by changing caterers?

Figure 6.7

Solution

Step 1: Cost per employee using Yummy Foods = $25.00 ÷ 55 + $3.50 = $0.46 + $3.50 = $3.96

Step 2: Cost per employee using Greedy Gourmet = $40.00 ÷ 55 + $3.00 = $0.73 + $3.00 = $3.73

Step 3: Cost savings = $3.95 − $3.73 = $0.22

The three separate problems in Figure 6.7 are (1) to find the cost per employee using Yummy Foods, (2) to find the cost per employee if using Greedy Gourmet, and (3) to find the difference between the two costs.

TROUBLESHOOTING PROBLEMS

Level 6 introduces troubleshooting problems. Problems involving Level 3 to Level 5 skills should be presented to learners with incorrect solutions. Learners should be able to determine either exactly where an error was made or the difference between the incorrect solution and the correct one, as in the case of an incorrect cost or payment (see Figure 6.8).

> A customer in the store where you work purchases items costing $3.50, $4.95, and $5.25, and pays with a $20 bill. You give the customer $7.40 in change. How much, if at all, did you overcharge or undercharge the customer?

Figure 6.8

Solution

Step 1: $3.50 + $4.95 + $5.25 = $13.70

Step 2: Change should be $20.00 − $13.70 = $6.30

Step 3: You gave $7.40 − $6.30 = $1.10 too much change; therefore, you undercharged the customer.

Troubleshooting skills are crucial on the job. Salesclerks often deal with customers who have been overcharged or undercharged, as in Figure 6.8. Many mistakes cost businesses money. Understanding where the mistake was made can help prevent its recurrence in the future. Learners should develop their own checklists that include common errors to look for. These checklists could start with the following questions:

♦ Were calculations done using numbers all converted to the same units?

♦ Were all decimals placed correctly?

♦ Were operations (e.g., addition, subtraction) performed correctly or, if a calculator was used, were numbers entered correctly?

♦ Were operations performed in the correct order (e.g., a discount applies to only one of several items a customer is purchasing and should be taken off before all items are totaled)?

♦ Was the correct formula used and/or were values substituted appropriately (e.g., radius instead of diameter)?

♦ If fractions were used, were common denominators found or inversions done when necessary?

♦ Were conversions of percents done correctly?

♦ Were operations on mixed units, or denominate numbers, done correctly?

OTHER STRATEGIES FOR IMPROVING SKILLS TO LEVEL 6

Have learners calculate the amount of carpet or flooring needed for rectangular rooms. In the workplace, Level 6 problems may involve figuring installation charges separately or subtracting a discount.

Making scale drawings such as room arrangements, buildings, vehicles, and landscape designs provides good practice in using ratios and proportions. This activity also provides an opportunity to discuss the results of not adhering to the same scale or proportion.

Determining the amount of cement needed for a sidewalk or driveway is a common volume problem. It can also involve converting units. Although cement is sold by the cubic yard, the thickness of a sidewalk is usually measured in inches, and the width and length are measured in feet. Have learners find, for example, the number of cubic yards of cement needed for a sidewalk that is to be 4 inches thick, 3 feet wide, and 40 feet long.

Have learners work with multiplication and division of fractions by increasing recipes or measurements for lumber.

Rate problems can be based on learners' personal experiences in their own jobs. Learners could begin with amounts of material and time used to produce one item and determine the amount of material needed for a day's or week's production.

"Trick problems" are often good exercises in eliminating extraneous information. A classic example is the nursery rhyme "As I Was Going to St. Ives":

> As I was going to St. Ives
> I met a man with seven wives
> Each wife had seven sacks
> Each sack had seven cats
> Each cat had seven kits
> Kits, cats, sacks, wives—
> How many were going to St. Ives?

The extraneous information in this rhyme is so overwhelming that the reader easily overlooks what the question is really looking for (only one person was going *to* St. Ives; the rest were going the other way). There are many newer versions of this type of problem that present a fairly simple question buried in a great deal of information.

Level 6 Sample Items

Problem 6.1

You need to haul a load of patio bricks to a job site. Each brick weighs 4 pounds 14 ounces. Your truck can carry a $\frac{3}{4}$-ton load. How many bricks can your truck carry in a full load?

- **A.** 300
- **B.** 307
- **C.** 362
- **D.** 409
- **E.** 483

Solution

Step 1: 4 pounds 14 ounces = 4 pounds + 14 ounces ÷ 16 ounces/pound = 4.875 pounds

Step 2: $\frac{3}{4}$-ton × 2,000 pounds/ton = 1,500 pounds

Step 3: 1,500 pounds ÷ 4.875 pounds/brick = 307.69 bricks rounded down to 307 bricks

Indications that this is a Level 6 problem:

♦ Calculations involve mixed numbers and fractions.

♦ One-step conversions are necessary.

♦ This is a multiple-step problem with several steps of reasoning.

The number of bricks must be rounded down because if it were rounded up, the weight would be more than a full load.

Problem 6.2

$ The catering service where you work employs 4 people whose individual hourly wages are $12.20, $13.25, $14.45, and $16.00, respectively. They each work 8 hours per day, 5 days per week, and are paid for all holidays plus 2 weeks of vacation per year. What is the total amount of the annual payroll?

    A. $ 3,354
    B. $ 14,534
    C. $ 26,832
    D. $116,272
    E. $120,744

Solution

Step 1: Total pay per hour = $12.20 + $13.25 + $14.45 + $16.00 = $55.90

Step 2: Hours per year = 8 hours/day × 5 days/week × 52 weeks/year = 2,080 hours/year

Step 3: Total pay = 2,080 hours/year × $55.90/hour = $116,272 per year

Indications that this is a Level 6 problem:

♦ Learners must set up the problem and do several steps of calculation and conversion.

♦ Measurement conversions are necessary.

Learners should understand the term *annual payroll.* They can calculate the number of working hours in a year and multiply this number times the total pay per hour. It is assumed that they know how many weeks there are in a year.

7 Description of Level 7 Skills

At Level 7, the task may be presented in an unusual format and the information presented may be incomplete or implicit. Tasks often involve multiple steps of logic and calculation. In addition to demonstrating the skills at the previous levels, individuals with Level 7 skills can:

■ **Solve problems that include nonlinear functions (such as rate of change) and/or that involve more than one unknown.** In the workplace, Level 7 problems that use more than one unknown are tasks that commonly require combining rates or combining mixtures.

■ **Convert between systems of measurement that involve fractions, mixed numbers, decimals, and/or percentages.** Individuals with Level 7 skills are able to convert 7 pounds $5\frac{1}{2}$ ounces to grams or kilograms.

■ **Calculate volumes of spheres, cylinders, or cones.** At Level 7, individuals are able to determine volumes of various shapes, given the formulas.

■ **Calculate multiple areas and volumes.** At Level 7, individuals deal with problems involving irregular shapes. In order to find total area or volume, these shapes should be segmented into regular shapes for which the individual can find the areas.

■ **Set up and manipulate complex ratios or proportions.** At this level, individuals encounter ratios that can contain mixed units requiring conversions. Individuals may need to do other calculations before or after setting up the proportion.

■ **Find the best deal when they have several choices.** Individuals with Level 7 skills are able to determine the best economic value of several alternatives by using graphics or by finding a percentage difference or a unit cost.

■ **Find mistakes in Level 6 problems.** At Level 7, individuals are able to check for errors in Level 3 to Level 6 problems.

■ **Apply basic statistical concepts.** Level 7 skills enable a person to find weighted averages and use median and mode to compare quantities.

Moving to Level 7 Skills

Level 7 problems can include extraneous information, many steps of reasoning, troubleshooting, best deal calculations, and any other skills discussed at the earlier levels in any combination. Emphasis on careful reading and reasoning is particularly important.

MULTIPLE STEPS AND CALCULATIONS

In the problem-solving system described at Level 3, the first step is to decide exactly what the question is. When setting up an equation, this translates into identifying the unknown and then choosing a variable to represent that unknown value. The second step, determining the facts, is the process of identifying what is known. Finally, choosing the correct operations and the order in which they should be performed is pertinent to actually setting up the equation. Figure 7.1 was originally a Level 6 problem until a complication (the 2% shrinkage) was added:

You must figure the amount of fabric necessary to make a drape for an office window. The finished drape must be $7\frac{1}{2}$ feet long and 5 feet wide. The fabric is wide enough to allow for side hems, but you must determine the length needed. You need to allow 8 inches extra at the top and at the bottom to fold under for the hems. You must also allow an extra 2% in length for possible shrinkage. How many linear yards of fabric should you need to make the drape?

Figure 7.1

Solution

Step 1: $7\frac{1}{2}$ feet × 12 inches/foot = 90 inches

Step 2: 90 inches + 8 inches + 8 inches = 106 inches

Step 3: 106 inches ÷ 12 inches/foot ÷ 3 feet/yard = 2.94 yards

Step 4: 2.94 yards × 2% = 2.94 yards × 0.02 = 0.06 yards

Step 5: Total yards needed = 2.94 yards + 0.06 yards = 3.00 yards

An implicit factor is that the 2% shrinkage allowance applies to all of the other measurements of length. Obviously, some conversions between systems are necessary before the calculations can be done.

NONLINEAR FUNCTIONS

Although Level 7 skills do not include setting up or solving nonlinear equations, learners should be able to use graphs, tables, and formulas representing this type of function. Problems involving sales commissions, income taxes, and other nonlinear functions found in the workplace may appear at this level. Sample Problem 7.1 (see page 39), which deals with bottle cap production, is an example of a problem based on a graph of a nonlinear function.

MORE THAN ONE UNKNOWN

Learners need to understand that problems with more than one unknown can be approached in one of two ways. In the first method, both unknowns are represented in terms of one variable. This variable represents one of the unknowns, and the other unknown is represented by a known relationship between the unknowns using the same variable. The second method for solving problems with more than one unknown uses two different variables and two equations. The problem in Figure 7.2 is an example of using more than one unknown.

You must determine the ticket prices for an upcoming performance in the concert hall where you work. You have been told that the gross sales, if all the tickets are sold, should total $14,000. There are 400 first-floor seats and 200 balcony seats in the hall. The first-floor tickets are to be priced at $5 more than the balcony seats. What price should you charge for each type of ticket?

Figure 7.2

Solution

One unknown:

Step 1: Let X represent the balcony ticket price.

Step 2: Let X + $5 represent the first-floor tickets.

Step 3: $200X + 400(X + \$5) = \$14,000$

Step 4: $200X + 400X + \$2,000 = \$14,000$

Step 5: $600X + \$2,000 = \$14,000$

Step 6: $600X = \$14,000 - \$2,000$

Step 7: $600X = \$12,000$

Step 8: $X = \dfrac{\$12,000}{600}$

Step 9: $X = \$20$

Step 10: Balcony tickets are $20, so the first-floor tickets are $20 + $5 = $25.

Two unknowns:

Step 1: Let X represent the balcony ticket price.

Step 2: Let Y represent the first-floor ticket price.

Step 3: $Y = X + \$5$

Step 4: $200X + 400Y = \$14,000$

Step 5: $200X + 400(X + \$5) = \$14,000$

Step 6: $200X + 400X + \$2,000 = \$14,000$

Step 7: $600X + \$2,000 = \$14,000$

Step 8: $600X = \$14,000 - \$2,000$

Step 9: $X = \dfrac{\$12,000}{600}$

Step 10: $X = \$20$

Step 11: $Y = X + \$5 = \$20 + \$5 = \$25$

PERCENTAGE

Percentage is often used to show a change in a quantity or value. Most problems calculating percent of change are at least two-step problems. First, the amount of change should be determined if it is not already known. Then the amount of change is divided by the original value (see Figure 7.3).

You are a member of a committee that must prepare a request for pay raises at your company. In the past three years, the average pay of the workers has gone from $12.50 per hour to $13.75 per hour. During the same time, the cost of living in your area has increased 12.5%. What percent of change occurred in the average pay in this time period?

Figure 7.3

Solution

Step 1: Change = $13.75 − $12.50 = $1.25

Step 2: Percent change = $\frac{\$1.25}{\$12.50}$ = 0.10 = 10%

The cost-of-living information, while undoubtedly very pertinent to the committee, is extraneous to this specific question. Many statistics provide material for practicing the calculation of percent change. Changes in temperature, prices, sales, production, plant capacity, and demand are all sources of possible problems.

MULTIPLE AREAS

Many workplace problems involve calculating an area that is not one simple geometric figure. These irregular figures can usually be broken down into standard figures that fit standard formulas. For instance, an L-shaped room that is being measured for carpet can be divided into two rectangles in order to determine the area. This process is called figuring a combined or multiple area. Or, if the amount of paint required for a rectangular wall with two windows and a door needs to be determined, the areas of the windows and door can be subtracted from the total area of the wall. This is called figuring a reduced area.

VOLUME

Some geometric problems at Level 7 involve finding the volume of spheres, cylinders, or cones. Sample Problem 7.4 (see page 42) provides an example of a problem that requires finding the volume of a sphere. The radius of the bowling ball is 4 inches, and this is the only information learners need to complete the problem. You may want to review the use of exponents, which were introduced at Level 5. In the case of this problem, 4^3 means $4 \times 4 \times 4$, not 4×3.

Figure 7.4 is a cylindrical volume problem; the measurement given, 2.5 feet across on the inside, is the diameter, not the radius. The other point that may be confusing in this problem is the conversion of units. Because this is a change from a larger to a smaller unit, it is necessary to multiply. Again, careful reading of the information is the key to these problems.

The farm where you just started working has a cylindrical oil tank that is 2.5 feet across on the inside. The depth of the oil in the tank is 2 feet. If 1 cubic foot of space holds 7.48 gallons, about how many gallons of oil are in the tank?

Figure 7.4

Solution

Step 1: Find the radius: $r = \frac{d}{2} = \frac{2.5 \text{ feet}}{2} = 1.25$ feet

Step 2: Find the volume:
$v = \pi r^2 h = 3.14 \times (1.25 \text{ feet})^2 \times 2 \text{ feet} = 9.8 \text{ feet}^3$

Step 3: 9.8 feet^3 × 7.48 gallons/feet^3 = 73.3 gallons

RATIOS AND PROPORTIONS

Some problems found at Level 7 require manipulation of complex ratios and proportions. These may contain a mixture of fractions and decimals, and differing units of measurement.

> You can install 12 square yards of flooring in 3 hours and 15 minutes. At this rate, about how long should it take you to put the same type of flooring in a room that is 10 feet 4 inches by 15 feet 9 inches?

Figure 7.5

Solution
Step 1: Find the room area:

$A = lw = 15$ feet 9 inches $\times$ 10 feet 4 inches

$A = 15.75$ feet $\times$ 10.33 feet $= 162.7$ feet^2

Step 2: Convert to square yards:

162.7 feet^2 ÷ 9 feet^2/yard^2 = 18.1 yard^2

Step 3: Set up proportion:

$$\frac{3 \text{ hours 15 minutes}}{12 \text{ square yards}} = \frac{X}{18.1 \text{ square yards}}$$

Step 4: Solve proportion:

$$X = \frac{3 \text{ hours 15 minutes}}{12 \text{ square yards}} \times 18.1 \text{ square yards} =$$

$$\frac{3.25 \text{ hours}}{12 \text{ square yards}} \times 18.1 \text{ square yards} = 4.90 \text{ hours}$$

Step 5: Convert time:

4.90 hours = 4 hours 54 minutes

To solve the problem in Figure 7.5, a proportion is set up in the same manner as the simpler ones found in the previous levels. Learners should notice that there are several possible proportions that could be used in this instance. The operations needed are more complex because of the mixed units and necessary conversions. The area of the room should be found and the value changed to square yards. To use a calculator, the 3 hours and 15 minutes can be changed to a decimal or to minutes.

In the problem found in Figure 7.6, the learner must do conversions, set up a proportion to find the amount of supplement, determine if one or both of the discounts apply to this sale, and find the total cost. The 45% figure describing the supplement is extraneous, as is the second discount in this particular case.

> In your agricultural feed sales job, you must place an order for a feed supplement for pigs. You have already determined that the proper ratio in this case is 1,000 lbs. of corn and 700 lbs. of oats to 300 lbs. of 45% grower supplement. The farmer will combine these components in a mixer with a $1\frac{1}{2}$-ton capacity and wants to order enough 45% grower supplement to mix 8 batches. The supplement sells for $260 per ton. Partial tons are figured at 14¢ per pound. If the order is for more than one full ton, there is a $20 discount on each full ton. If the order is for more than 2 full tons, there is an additional $5 discount on each ton. How much should you charge the farmer for this order of supplement?

Figure 7.6

Solution
Step 1: Total ratio weight = 1,000 pounds + 700 pounds + 300 pounds = 2,000 pounds = 1 ton

Step 2: Total weight of batches = $1\frac{1}{2}$ tons/batch $\times$ 8 batches = 12 tons

Step 3: Set up proportion:

$$\frac{300 \text{ pounds supplement}}{1 \text{ ton}} = \frac{X}{12 \text{ tons}}$$

Step 4: Solve proportion:

$$X = \frac{300 \text{ pounds supplement}}{1 \text{ ton}} \times 12 \text{ tons} =$$

3,600 pounds = 1 ton 1,600 pounds

Step 5: Determine price:

1 ton $\times$ ($260 − $20) = $240

1,600 pounds $\times$ $0.14/pound = $224

Total price = $240 + $224 = $464

BEST DEAL

At Level 7, the best deal problems include calculating the possible economic values, finding the difference, finding the percent of difference, and determining the unit cost for the best deal.

> The flower shop where you work uses large spools of wide outdoor ribbon for Memorial Day arrangements. Flowersupply, your regular supplier, sells the ribbon in individual spools for $7.90 each, or for $7.20 each if you buy them in full lots of 20 spools. Beribboned Company offers the same ribbon at $87.00 for a dozen spools. If you need to order 36 spools of ribbon, what percent can you save by going with the lower price?

Figure 7.7

Solution

Step 1: Calculate the two prices:
Flowersupply: 20 spools × $7.20/spool + 16 spools × $7.90/spool = $144 + $126.40 = $270.40
Beribboned Company: $87/dozen × 3 dozen = $261

Step 2: Calculate savings: $270.40 − $261 = $9.40

Step 3: Calculate percent savings: $\frac{\$9.40}{\$270.40} \times 100\% = 3.48\%$

The problem asks what percent is saved; it is implicit that the savings would be on the higher price. To find the percent of savings, the difference is divided by the higher price. The result is expressed as a savings of 3 percent. This kind of problem could instead be set up to find the unit cost per spool of ribbon from each supplier. Or the problem could include shipping costs for one or both suppliers to be figured into the total cost.

TROUBLESHOOTING

The troubleshooting problems encountered at Level 7 ask learners to find mistakes in multiple-step calculations. Learners may be required to find the correct answer or to pinpoint where an error was made. When the problem asks for the correct answer only, all that is necessary is to refigure the problem. If the problem asks where the mistake was made, more analysis is required. For example, the problem in Figure 7.5 might be presented in the following manner:

> You can install 12 square yards of flooring in 3 hours and 15 minutes. You need to put the same type of flooring in a room that is 10 feet 4 inches by 15 feet 9 inches and figure it will take you 41 hours and 6 minutes. You suspect, based on estimation, that your calculations are wrong. What error, if any, did you make?

Figure 7.8

First, learners should check whether there was an error in entering numbers in the calculator. Next, they should consider the other skills involved in the problem:

◆ Was the proportion set up correctly?

◆ Were all conversions completed and done correctly?

◆ Were all operations done and the correct operations used?

◆ Were there any errors in the placement of decimal points?

In this case, once the square footage of the room was calculated, it was not converted to square yards. In some cases, several trials may be necessary in order to find the error. In others, estimation may help point to the problem. In the case of Figure 7.8, learners may consider that 12 square yards could be the area of a room 9 feet by 12 feet. The room to be floored is less than twice as large, so an appropriate estimate of time would be about 5 hours. The answer arrived at was 8 to 9 times that amount. Since there are 9 square feet to a square yard, this observation could indicate the difficulty.

OTHER STRATEGIES FOR IMPROVING SKILLS TO LEVEL 7

$ Develop long-range simulations centering on a specific fictional or real business. Learners can do problems involving measurement, inventory, retail sales, or any other facet of this business. Business math textbooks for high school and postsecondary classes might be helpful here.

$ Have learners conduct projects in which they collect information that would be useful in starting a new business.

1 2 Use geometric math puzzles to practice identifying regular shapes.

1 2 Have learners use standard containers, from canned goods to barrels, to practice figuring volume.

% Use statistics on population, sales, and other data involving change to develop problems that learners can use to figure percent of change.

% Develop best deal problems such as those suggested in the strategies at Level 5, and use the resulting answers in other problems (e.g., finding the percent of discount, unit cost for each, and cost per unit weight or volume).

Level 7 Sample Items

Problem 7.1

% **#** You operate a machine that stamps bottle caps out of 3-inch-by-3-inch aluminum squares. Occasionally, the machine produces an unusable cap, a reject, that must be recycled. The number of rejects made at different production rates is shown below. Today you have been told to produce 600 caps per hour. Approximately how many caps total should you have to produce to end up with your quota of 2,400 good ones?
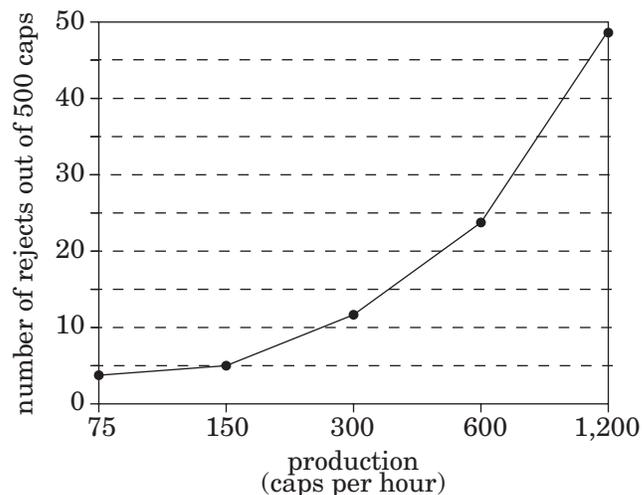
A. 2,400
B. 2,448
C. 2,521
D. 2,548
E. 2,616

Solution

Step 1: Reject rate at 600 caps per hour from the graph: 24 bad caps per 500 caps

Step 2: Acceptance rate: 500 caps – 24 caps = 476 good caps per 500 caps

Step 3: Set up proportion: $\dfrac{476}{500} = \dfrac{2,400}{X}$

Step 4: Solve for X: $\dfrac{476}{500} = \dfrac{2,400}{X}$;

$$X = \frac{2,400 \times 500}{476} = 2,521 \text{ caps}$$

♦ Learners must do several steps of reasoning and calculation.

♦ Learners must set up and manipulate a proportion by interpreting information from a graph.

This problem requires first an exact understanding of what the question is asking: How many caps, in addition to the 2,400 that you need, must be produced in order to cover the number of rejects at that production rate? By carefully examining the graph, the learner should find that at 600 caps per hour, 24 rejects are produced out of every 500 caps, or 476 good caps per 500 are produced at this rate. Other than finding the number of rejects per hour, the 600 rate is extraneous because time is not a factor here.

Problem 7.2

Your job with the Department of Parks and Recreation requires you to plant *Ginkgo* trees along the streets of your city. You have 500 of the trees that are ready for planting. You are to plant the trees along the blocks as shown by the dashed lines on the map below. The trees must be planted 30 feet apart. Each block is $\frac{3}{8}$ mile long. How many more trees, if any, do you need to complete the job?
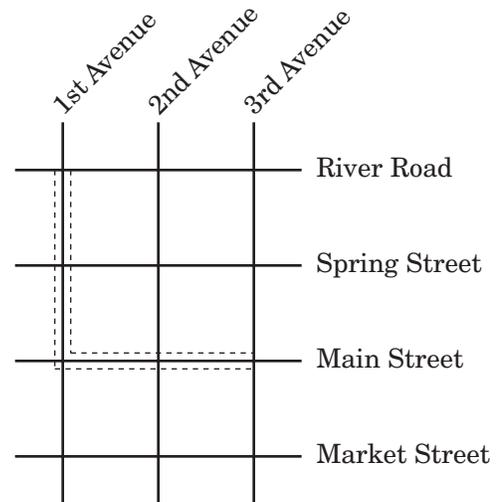
A.   None; you have 236 extra trees.
B.   None; you have exactly enough.
C.   You need 20 more trees.
D.   You need 908 more trees.
E.   You need 15,340 more trees.

Solution

Step 1:   Convert miles to feet:

$$\frac{3}{8} \text{ mile} \times \frac{5{,}280 \text{ feet}}{1 \text{ mile}} = 1{,}980 \text{ feet}$$

Step 2:   Find the number of trees:

$$1{,}980 \text{ feet} \div \frac{30 \text{ feet}}{\text{tree}} = 66 \text{ trees}$$

Step 3:   Correct for street: 66 trees − 1 tree = 65 trees

Step 4:   Calculate total trees:

$$8 \text{ blocks} \times \frac{65 \text{ trees}}{\text{block}} = 520 \text{ trees}$$

Step 5:   Correct for intersection:

520 trees + 1 tree − 1 tree = 520 trees

Step 6:   Calculate trees needed:

520 trees − 500 trees = 20 trees

In order to first find the number of trees needed for each block, $\frac{3}{8}$ of a mile needs to be converted to 1,980 feet. This number is then divided by 30, which results in 66. Since the length of a block is from the center of one cross street to the next and trees are not wanted in the middle of the streets, only 65 trees are needed in each block. There are 8 blocks, considering both sides of the street; therefore, $8 \times 65 = 520$ are needed. However, there are two other factors to be taken into consideration. The tree placements at the northeast corner of the intersection of 1st Avenue and Main Street intersect resulting in two trees at the same place, so one tree less would be needed. However, the dashed lines on the map show a tree is needed at the southwest corner of that same intersection. The two factors cancel each other out, so 520 trees are needed.

Indications that this is a Level 7 problem:

♦ Learners must do several steps of reasoning and calculation.

♦ Learners must figure out the information needed to solve the problem when the information presented is incomplete or implied.

♦ Learners must convert between systems of numbers that involve fractions.

Problem 7.3

You have to order fencing for a 25-acre, rectangular field. One side of the field measures exactly $\frac{1}{4}$ mile. How many yards of fencing will you need to enclose the field completely?

A. 1,320
B. 1,430
C. 4,290
D. 363,000
E. 1,089,000

Solution

Step 1: Understand that: 1 acre = 43,560 square feet

Step 2: Convert acres to square feet: 43,560 square feet/acre × 25 acres = 1,089,000 square feet

Step 3: Convert miles to feet: 1 mile = 5,280 feet

Step 4: Calculate width of field in feet:
5,280 feet/mile × 0.25 mile = 1,320 feet

Step 5: Compute length of field in feet:
1,089,000 square feet ÷ 1,320 feet = 825 feet

Step 6: Compute perimeter using formula:
(2 × 825 feet) + (2 × 1,320 feet) = 4,290 feet

Step 7: Convert to yards:
4,290 feet ÷ 3 feet/yard = 1,430 yards

Indications that this is a Level 7 problem:

♦ Learners must do several steps of reasoning and calculation.

♦ Learners must convert between systems of measurement that involve fractions, mixed numbers, decimals, or percentages.

♦ Learners must figure out the information needed to solve the problem when the information presented is incomplete or implied.

Problem 7.4

In your job at a company that makes bowling balls, you are doing a quick check of how much plastic resin has been used this week by the machine that molds the balls. Each ball is a sphere with a radius of 4 inches (the finger holes are drilled out after the ball has been molded). Production records show that 1,200 balls were made this week. Ignoring waste, about how many cubic feet of resin were used in the machine this week?

- A.   46
- B.   186
- C.   268
- D.   2,233
- E.   321,536

Solution

Step 1: Calculate volume of the ball:

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times (4 \text{ inches})^3 = 268 \text{ inches}^3$$

Step 2: Calculate total volume:

$$1,200 \text{ balls} \times 268 \text{ inches}^3 = 321,600 \text{ inches}^3$$

Step 3: Convert to cubic feet:

$$321,600 \text{ inches}^3 \div 1,728 \text{ inches}^3/\text{feet}^3 = 186 \text{ feet}^3$$

Indication that this is a Level 7 problem:

♦ Learners must calculate the volume of a sphere.

♦ Learners must convert units.

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APPENDIX A

Applied Mathematics Formula Sheet

Distance

1 foot = 12 inches

1 yard = 3 feet

1 mile = 5,280 feet

1 mile ≈ 1.61 kilometers

1 inch = 2.54 centimeters

1 foot = 0.3048 meters

1 meter = 1,000 millimeters

1 meter = 100 centimeters

1 kilometer = 1,000 meters

1 kilometer ≈ 0.62 miles

Area

1 square foot = 144 square inches

1 square yard = 9 square feet

1 acre = 43,560 square feet

Volume

1 cup = 8 fluid ounces

1 quart = 4 cups

1 gallon = 4 quarts

1 gallon = 231 cubic inches

1 liter ≈ 0.264 gallons

1 cubic foot = 1,728 cubic inches

1 cubic yard = 27 cubic feet

1 board foot = 1 inch by 12 inches by 12 inches

Weight

1 ounce ≈ 28.350 grams

1 pound = 16 ounces

1 pound ≈ 453.592 grams

1 milligram = 0.001 grams

1 kilogram = 1,000 grams

1 kilogram ≈ 2.2 pounds

1 ton = 2,000 pounds

Rectangle

perimeter = $2\,(length + width)$

area = $length \times width$

Rectangular Solid (Box)

volume = $length \times width \times height$

Cube

volume = $(length\ of\ side)^3$

Triangle

sum of angles = 180°

area = $\frac{1}{2}(base \times height)$

Circle

number of degrees in a circle = 360°

circumference ≈ $3.14 \times diameter$

area ≈ $3.14 \times (radius)^2$

Cylinder

volume ≈ $3.14 \times (radius)^2 \times height$

Cone

volume ≈ $\dfrac{3.14 \times (radius)^2 \times height}{3}$

Sphere (Ball)

volume ≈ $\frac{4}{3} \times 3.14 \times (radius)^3$

Electricity

1 kilowatt-hour = 1,000 watt-hours

amps = watts ÷ volts

Temperature

°C = $0.56\,(°F - 32)$ or $\frac{5}{9}(°F - 32)$

°F = $1.8\,(°C) + 32$ or $\left(\frac{9}{5} \times °C\right) + 32$

NOTE: Problems on the WorkKeys *Applied Mathematics* assessment should be worked using the formulas and conversions on this formula sheet.